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DISCUSSION AND CORRESPONDENCE

A REMARKABLE ECLIPSE

To the Editor of Science: Eclipses of the sun and moon occur with such frequency and are so similar in character and appearance that a distinction between them sufficiently great to be noticed by the uncritical observer would seem to be out of the question. The cause of eclipses is well known, and as they may be easily calculated the times of their occurrence and nature of their appearance are always published in the Nautical Almanac two or three years before they actually take place. Total eclipses of the sun have for many years afforded the necessary darkness for observing the heavens in close proximity to the sun; and numerous expeditions have been sent to distant parts of the earth in order to take advantage of the few moments of additional darkness thus afforded; and much interesting and useful information concerning the physical constitution of the sun has been obtained in this manner. At the present time, however, the chief importance of eclipses lies in the opportunities they afford for testing the accuracy of the calculations of mathematicians, and the correctness of the physical theories on which such calculations are based; and for this purpose the distinction between partial and total eclipses is of little importance.

In the year 1915 there were only two eclipses, both of the sun. The first occurred on February 13 under ordinary circumstances; the central eclipse began at sunrise in the Indian Ocean a few degrees to the southward of the island of Madagascar; passing along the northwestern coast of Australia, it crossed the island of New Guinea and ended at sunset in the North Pacific Ocean. The second eclipse took place on August 10; beginning at sunrise a few degrees to the southward of the Japanese Islands in the North Pacific Ocean. It moved to the eastward a few degrees southward of the Sandwich Islands at noon, and ended at sunset in the South Pacific Ocean. These two eclipses were very similar in character in so far as outward appearances are concerned. Their relative importance arises from the very dissimilar conditions under which they took place. In the eclipse of August 10 the centers of the *sun, moon* and *earth* were very nearly *in the same straight line.* I have examined the record of all the eclipses that have taken place since the year 1767; and I find that in the year 1903 there were two very similar eclipses; one of which took place on February 21 and the other on August 17 of that year.

It has, therefore, been *one hundred and twelve years* since a similar eclipse happened; and I find that the next similar eclipse will occur on July 11, 1991, or *seventy-six years from the present time.* It is, therefore, only on very rare occasions that such eclipses take place and this fact seems worthy of mention in the historical record of important eclipses.

It may, however, interest the reader to know how or why I happened to make this important discovery, as it has been many years since I was engaged in the discussion of eclipses for chronological purposes. I will, therefore, give a brief account of my investigations which so happily led to this discovery.

In the early summer of the year 1906 I was much embarrassed by a superfluity of leisure, and unable to pass my time agreeably with nothing to do. I had then recently been reading G. H. Darwin's interesting book on "The Tides and Kindred Phenomena," and learned that the mathematical theory of the tides was in a very unsatisfactory condition. I had read in my younger days the explanations of the tides by Newton and by Laplace. These explanations seemed so *plausible* that I then accepted them as correct. But as I had devoted the greater part of my life to the discussion of gravitational problems, the thought occurred to me that possibly a new discussion of an old problem might throw additional light upon a subject which was confessedly very obscure. I therefore concluded to undertake a critical discussion of the theory of the tides, and the discovery of the remarkable eclipse came as a bi-product of that discussion. My leisure has

since been pleasantly devoted to a study of the tides and other kindred problems.

In my investigation of the tidal problems I have based my work on the two following postulates; namely:

FIRST: *If a solid body of any figure whatever be covered by a fluid in equilibrium, the gravity at every point of the surface will be the same;* and

SECOND: *If the fluid covering a solid body be free to flow, and the gravity at different points of its surface be disturbed in any manner whatever, the fluid will flow from points where gravity is less to points where gravity is greater; and it will continue to flow until the gravity at all points of the surface becomes equal.*

If these postulates in regard to the equilibrium of fluids be correct the problem of the tides becomes greatly simplified, and instead of being the most *difficult,* it becomes the *simplest* problem of celestial mechanics. For it is a very simple problem to calculate just how much the earth's gravity at any point of its surface is affected by the attraction of the sun and moon. Now when the sun or moon is overhead we know the gravity at the earth's surface directly underneath them is lessened, and we also know that the gravity at all points where the sun or moon is in the horizon is increased by their attraction. It therefore follows from the second postulate that the water directly under the sun or moon will flow away towards the horizon in every direction; and instead of being heaped up under the moon as claimed by Newton and his successors, will be dispersed along a great circle of the earth whose pole is directly under the sun or moon, thus making a thin ribbon or narrow zone of high water of uniform depth and extending completely around the earth, instead of being piled up in the form of protuberance under the moon.

It also follows that there will be a zone of low water directly under the moon instead of a protuberance of high water as claimed by Newton.

Now since there are two disturbing bodies, the sun and the moon, acting independently of each other, it is evident that there will be two independent high-water waves passing completely around a great circle of the earth; and since all great circles intersect or cross each other at opposite extremities of a diameter, it follows that there will always be two points of intersection, or two places of high water, which may be observed at all times, provided we know where to look for them. It also follows that high tides are not restricted to the times of new and full moon, but exist at all times.

The problem of the tides is therefore greatly simplified and reduced to one of finding where the high-water waves produced by the attraction of the sun and moon cross each other, for at these points the single wave is equal to the sum of the two separate waves; and the computation of the places where the tidal waves cross each other is very easy and much simpler than the computation of an eclipse.

The plane of the solar tidal wave is always perpendicular to the ecliptic, and passes through the center of the earth and poles of the ecliptic; and its position is known at all times. The plane of the lunar tidal wave is always perpendicular to the plane of the moon's orbit; but as the moon's orbit is inclined to the ecliptic by about 5°, it follows that the poles of the moon's orbit are always at a distance of 5° from the poles of the ecliptic. But the inclination of the moon's orbit to the ecliptic is always the same, while the nodes of the orbit on the ecliptic are in motion, and perform a complete revolution in about nineteen years. It follows from this that the poles of the moon's orbit move in a small circle of 5° radius around the poles of the ecliptic, making a revolution in nineteen years. The position of the lunar tidal wave thus becomes known at all times; and since the position of the solar tidal wave is also known at the same time, it becomes an easy matter to calculate the place of their intersection, which is the place of high tide.

Now since the moon's nodes are moving backward on the ecliptic 1°.5649 during each lunation, it follows that the tides of no two consecutive lunations will be precisely the same; but they may be more easily calculated than most other celestial phenomena.

In the early spring of the present year (1915) I had so far completed the construction of mathematical formulas for the computation of the tides, that I actually computed the latitude at which the two tidal waves crossed each other at noon of each day during the lunation between May 13 and June 13. This calculation led to the discovery that, whatever may be the relative declinations of the sun and moon at the moment of conjunction or opposition in right ascension, the two tidal waves will always cross each other exactly at the equator; and at the distance of 90° both east and west from the meridian on which the sun and moon are situated. During this lunation the two tidal waves crossed each other at an angle which varied between 4° 20′ and 90°; and the latitudes at which they crossed each other were less than 40° during about *three* days, at the times of new and full moon; while during the twenty-six or twenty-seven remaining days of the lunation the high water was confined within the latitude of 45° and 70°, making a *typical* high-water zone of about 25° in breadth.

Now it will be remembered by readers who are familiar with tidal history, that both Newton and Laplace were greatly embarrassed by the fact that the highest tides did not occur at the time when the acting forces were the greatest, but about a day and a half later; and in order to explain this default of theory, they were obliged to *assume* the operation of *fictitious* or *imaginary causes.* The observations on which their theories were based were made in southern England or northern France, in latitudes in which the united tidal wave did not *usually* reach until about a day and a half *after* the time of new or full moon; and the reason it was not observed was not on account of its non-existence but because it was on duty in another place.

We shall now consider the united tidal wave during the lunation beginning with the full moon of July and ending with that of August. According to the data given in the *American Ephemeris,* the two tidal waves at the instant of conjunction on August 10 made an angle with each other amounting to only 43.8″; so that they were practically superposed the one upon the other throughout their whole extent and reaching entirely around the earth. But as the lunar tidal wave travels over the earth's surface about thirteen times as fast as the solar tidal wave, they soon part company near the equator, each wave revolving around its own polar axis; and at the end of a single day the latitude of the united tidal wave will be found at 61° 40′ and the two tidal waves will cross each other at an angle of about 11° 20′. The united tidal wave will then remain on or very near to the parallel of 62° of latitude until near the end of the lunation; and there would be a daily succession of uniformly high tides on that parallel of latitude during nearly a whole month.

We have thus far considered only that portion of the tidal waves which rises *above* the normal surface of the ocean; but the water can not rise at any given place on the earth's surface without an equivalent depression at some other point; and a correct theory of the tides will explain equally well all the conditions incident to their formation. Now we know that the earth's surface-gravity is *diminished* by the attraction of the sun and moon at all points of the surface that are less than 54° 44′ of angular distance from those bodies, and *increased* for all greater distances. It therefore follows that all fluids that are under the sun and moon and are free to flow, will flow *away* from the point directly under the sun or moon, instead of *towards* it, as required by the present accepted theory of the tides. This follows for two reasons: *First,* because the earth's gravity is greater in that direction, and, *second,* because the *tangential* forces of the sun and moon actually *push* all bodies in that direction. This is one of the most beautiful and interesting consequences arising from the gravitation of matter; for, were the manner of its action to be reversed, the earth would no longer be habitable by man or beast; for the sun would be hidden by a perpetual cloud by day, and the moon by night; and neither of the luminaries would be visible except at rare and uncertain intervals.

Sir John Herschel in his "Outlines of

Astronomy" has called attention to the *unexplained fact* that the *full moon* tends to disperse the clouds under it. This follows as a necessary consequence of gravitation; but it is not restricted to the *full moon*, but is in active operation at all times by both sun and moon. The fact is however most easily observed at the time when the sun is absent.

Incidentally we may mention that were the moon's orbit in the plane of the ecliptic, the eclipse conditions of the tenth of August would be mostly repeated at each new moon; but the tidal phenomena would be fundamentally different. In the supposed case the crossing of the two tidal waves would be constantly at the pole of the ecliptic during the whole lunation, and the high tides would be confined to the latitudes of the arctic and antarctic circles. If, at the same time, the earth's equator were shifted into the ecliptic, there would be a *constant elevation* of water at both poles of the earth, while all other places on the surface of the earth would have four simple tidal waves each day. The general problem of the height of the tidal wave at any time and place on the earth's surface can not be considered here, but tables for that purpose have already been computed, though still unpublished.

We see from this exposition of the subject that all the infinite variety of tidal phenomena are fully explained by the operation of the forces of gravitation as developed under existing conditions in the solar system. The eclipse of August 10 represents a case in which the forces of the sun and moon act in perfect harmony during a few minutes of time; but it recurs at such infrequent and uncertain intervals that nothing useful can be learned from a single performance unless there be some known theoretical connection with preceding and subsequent events. The problem of the tides, which has been aptly called the "*Riddle of the Ages*," and designated in despair by an ancient philosopher as "*the tomb of human curiosity*," may therefore now be considered as completely solved.

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CLEVELAND,
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ON THE DEGREE OF EXACTNESS OF THE GAMMA FUNCTION NECESSARY IN CURVE FITTING[1]

THE note by Mr. P. F. Everitt in a recent number of this journal[2] discussing an earlier note by the present writer[3] seems so likely to obscure the essential point and purpose for which the earlier note was written that it appears desirable to advert to the subject once more.

In practical biometric work the gamma function is *chiefly* (though of course not entirely) used in connection with the fitting of Pearson's skew frequency curves, where such function appears in the expression for y_0. In other words, the exactness of approximation to the gamma function in these cases can affect nothing but the calculation of the ordinates and areas of the fitted curve. The writer finds it difficult to conceive of such circumstances in the ordinary prosecution of practical statistical researches as would necessitate or warrant the calculation of the ordinates or areas of a frequency curve to more than two places of decimals. This being the case, it seemed desirable, in the earlier paper, to call attention to the fact that a quite sufficiently "exact" approximation to the values of the gamma functions could be made by simple interpolation in a table of log $\lfloor n$.

In order that the statistical worker may form his own judgment as to what degree of exactness in approximating the gamma function is necessary in calculating y_0, Table I. is presented. This table shows, for four different skew frequency curves, the change produced in y_0 by altering the logarithm of the term involving gamma functions by the following amounts: .0000001, .000001, .00001, .0001 and .001. The curves used for illustration are taken from Pearson's memoir "On the Mathematical Theory of Errors of Judgment, with Special Reference to the Personal Equation."[4]

The curve marked I. in the table is Pear-

[1] Papers from the Biological Laboratory of the Maine Agricultural Experiment Station, No. 90.

[2] SCIENCE, N. S., Vol. XLII., pp. 453–455, 1915.

[3] SCIENCE, N. S., Vol. XLI., pp. 506–507, 1915.

[4] *Phil. Trans.*, Vol. 198*A*, pp. 235–299, 1902.